Graph Polynomial and Vertex Coloring

Abstract: Let G be a (simple) graph with edge set E and vertex set V={1,2,…,n}. The graph polynomial of G is

$$f_G(\vec{x}) = \prod_{ij\in E, i<j} (x_i - x_j) = \sum_{\vec{d}} C(\vec{d}) x_1^{d_1} x_2^{d_2} \cdots x_n^{d_n},$$

where $\vec{d} = (d_1, d_2, \cdots, d_n)$ satisfies $d_1 + d_2 + \cdots + d_n = |E|$.

It is clear that $\vec{c}: V \to R$ is a proper coloring if and only if $f_G(\vec{c}) \neq 0$.

It is also known that $|C(\vec{d})| = |EE(D) - EO(D)|$, where D is any orientation of E with out-degree sequence $\vec{d}$, EE(D) and EO(D) are the number of Eulerian subgraphs with even and odd number of edges, respectively. In this talk we will give a proof of Matiyasevich's criterion about k-colorability of G based on the number of Eulerian subgraphs (mod k).