MATH 1119B - Tutorial 1

0. This is an *augmented* matrix of a system of linear equations that is already fully reduced:

$$\begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & 4 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

   i. What is the size of the augmented matrix? The coefficient matrix?

   ii. How many equations does the system contain? How many unknowns/variables?

   iii. Convince yourself, as a group, why the system has an infinite number of solutions.
   Advanced: Can you tell me what all of the solutions look like?

1. Solve the system of equations

$$\begin{array}{rcrcrcr} 2x & - & 4y & + & z & = & 4 \\ -x & & & - & 2z & = & 2 \\ & - & 3y & + & 6z & = & 6 \end{array}$$

by

   i. Writing both the coefficient matrix and the augmented matrix of the system.

   ii. Row reduce the augmented matrix into the form

$$\begin{bmatrix} * & * & * & * \\ 0 & * & * & * \\ 0 & 0 & * & * \end{bmatrix}.$$

   This is called *row-echelon form (REF)*.

   iii. Use back-substitution to give the solution to the system, if any.

   iv. Further reduce the matrix into the form

$$\begin{bmatrix} 1 & 0 & 0 & * \\ 0 & 1 & 0 & * \\ 0 & 0 & 1 & * \end{bmatrix}.$$

   This is called *reduced row-echelon form (RREF)*. Compare your answer with that of [iii.].

2. Show the existence and uniqueness of the solutions to the systems represented by the following *augmented* matrices.

$$1.\ \begin{bmatrix} 1 & 1 & 1 & 2 \\ 2 & 2 & 2 & 3 \\ 3 & 1 & 4 & 2 \\ 4 & 1 & 2 & 3 \end{bmatrix}, \quad 2.\ \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 4 \end{bmatrix}, \quad 3.\ \begin{bmatrix} 2 & -4 & 1 & 4 \\ -1 & 0 & -2 & 2 \\ 0 & -3 & 6 & 6 \end{bmatrix}$$

3. Determine for which h and k the following augmented matrices give

   1. no solution,

   2. infinite solutions,

   3. unique solution.

$$\begin{bmatrix} 1 & -2 & 4 \\ 2 & h & k \end{bmatrix}, \quad \textbf{Advanced: } \begin{bmatrix} 1 & 2 & 4 \\ 0 & 2 & 6 \\ 0 & 2h & 3k \end{bmatrix}$$