Turkey Tutorial: MATH 1119B

This is for practice only, not for marks! Do not hand this in!
Answers (though perhaps not full solutions) will be provided eventually.
Please try the problems before seeing the solutions!
Have a good long weekend!
(Don't ask me why almost every sentence ends with a '!'!)

1. Identify if the following represent vector equations or matrix equations. If it is a vector equation, re-write it as a matrix equation. If it is a matrix equation, re-write it as a vector equation. If the equation is pre-solved for you, check that it is true. If the equation contains unknowns, solve it!

$$(a)\quad \begin{bmatrix} 1 & 1 & -1 \\ 2 & -3 & 4/3 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} = \begin{bmatrix} -4 \\ 12 \end{bmatrix}, \quad (b)\quad 2\begin{bmatrix} 1 \\ 2 \end{bmatrix} - 3\begin{bmatrix} 3 \\ -2 \end{bmatrix} + 3\begin{bmatrix} 12 \\ -3 \end{bmatrix} = \begin{bmatrix} 29 \\ 4 \end{bmatrix},$$

$$(c)\quad \begin{bmatrix} 2 & -2 & 3 \\ 1 & 2 & -1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \\ 8 \end{bmatrix}, \quad (d)\quad s_1\begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix} - s_2\begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix} + s_3\begin{bmatrix} 2 \\ -5 \\ 12 \end{bmatrix} = \begin{bmatrix} 8 \\ -6 \\ 4 \end{bmatrix}.$$

2. **Leontief exchange model** (first part of Section 1.6)
Ancient Greece is a very primitive place. Its economy has only two sectors: politics and military, and they each consume 50% of the total output. The following table shows the distribution of the input and output of Ancient Greece's economy:

Politics	Military	Purchased By:
.5	.5	Politics
.5	.5	Military

(i) Explain why each of the columns *must* add up to 1 (in any example!).
(ii) Give a system of equations which gives the output of a sector of the economy in terms of the inputs.
(iii) Re-arrange the equations to show that the system is *homogeneous*. Solve the system.
(iv) If Politics in the year 300 BC accounted for 10 thousand gold pieces, show the equilibrium point in each sector for the year 299 BC.
(v) **Advanced.** This example is simple. Can you see how, if the economy is split into only 2 sectors, this is the *only* table of input and output that gives a non-trivial equilibrium solution?
(vi) **Note:** I fixed the lecture slides for W4L2. Please review the example given there again. I *strongly* recommend that you do it by hand.

3. Recall that the *Span* of a set of vectors is defined as the set of *all linear combinations of the vectors*. That is, $\text{Span}(v_1, v_2, \ldots, v_n) = \{c_1v_1 + c_2v_2 + \cdots + c_nv_n, c_1, c_2, \ldots, c_n \in \mathbb{R}\}$. Picking different values for $c_1, c_2, \ldots, c_n$ provide different linear combinations, and thus different vectors.

Let $v_1 = \begin{bmatrix} 4 \\ 5 \\ -6 \end{bmatrix}$, $v_2 = \begin{bmatrix} -3 \\ -2 \\ 2 \end{bmatrix}$ and $v_3 = \begin{bmatrix} 1 \\ 5 \\ -3 \end{bmatrix}$. Set $b = \begin{bmatrix} -7 \\ -3 \\ 10 \end{bmatrix}$.

(i) Determine if b can be written as a linear combination of v_1, v_2 and v_3. That is, write $b = x_1v_1 + x_2v_2 + x_3v_3$, for some scalars x_1, x_2 and x_3.
(ii) Give a matrix A and a vector x such that x is the solution of the matrix equation $Ax = b$.
(iii) Determine if b is in the *Span* of v_1, v_2 and v_3.
(iv) Repeat (i)-(iii) with the vectors Let $w_1 = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$, $w_2 = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$ and $w_3 = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$ and $d = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.