# Math 1119B, Tutorial 5

Monday, November 7, 2011

**1.** Over time, my accounts department has noticed that proportions of accounts migrate at a constant rate. In a given year, 10% of my accounts which are typically paid on time become overdue by 30 days, and 5% of my accounts which are typically paid on time become overdue by 60 days or more. Similarly, 50% of accounts which are typically 30 days overdue are paid on time and 20% of accounts typically 30 days overdue become 60 or more days overdue. Finally, 10% of accounts which are typically 60 days overdue are paid on time, and 20% of these accounts become only 30 days overdue.

(a) Give a migration matrix which models this as a difference equation.

(b) My accounts vector in 2009 was

$$\begin{bmatrix} \text{on time} \\ \text{30 days overdue} \\ \text{60 or more} \end{bmatrix} = \begin{bmatrix} 1,000,000 \\ 600,000 \\ 300,000 \end{bmatrix}.$$

Determine what my accounts will be in 2011.

**2.** Let $A = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 12 & 15 \\ -1 & 3 & -5 \end{bmatrix}$.

(a) Determine if $A$ is invertible and, if so, find $A^{-1}$.

(b) If $b = \begin{bmatrix} 30 \\ -30 \\ 30 \end{bmatrix}$, find a solution to $Ax = b$.

**3.** Determine if the following transformations are linear. If they are linear, construct the associated matrix $A$ such that $T(x) = Ax$.

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}\right) = \begin{bmatrix} x_1 + x_2 + 2 \\ x_2 \\ x_3 - 2x_4 \\ 0 \end{bmatrix}, \qquad (b)S\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_3 \\ x_1 - x_2 \\ x_1 - x_2 \end{bmatrix}, \qquad R\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - x - 2 \\ x_1^2 - 2x_2 + 1 \end{bmatrix}.$$

**4.** Let

$$A = \begin{bmatrix} 1 & -1 \\ 2 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & -1 \\ 2 & -3 \end{bmatrix}, \quad C = \begin{bmatrix} 5 & -2 \\ 2 & 3 \end{bmatrix}.$$

(a) Find $\det(A), \det(B), \det(C)$.

(b) Find $(CB^T)^{-1} - A^2$.