Math 1119B, Tutorial 5

Monday, November 21, 2011

1. Find the determinants of the following matrices by **cofactor expansion**.

$$W = \begin{bmatrix} 3 & 5 & 7 \\ 0 & 1 & 2 \\ 2 & 0 & 1 \end{bmatrix}, \quad S = \begin{bmatrix} 2 & 3 & 5 & -7 & -9 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 2 & 3 & 0 & 5 \\ 2 & -1 & 12 & 0 & 0 \\ 0 & 0 & 3 & -1 & -3 \end{bmatrix}.$$

2. Find the determinants of the following matrices by **row-reduction** into upper-triangular form.

$$D = \begin{bmatrix} 2 & 1 & -3 \\ 3 & 0 & -4 \\ 1 & 2 & -1 \end{bmatrix}, \quad E = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 4 & 1 & 2 \\ 4 & 3 & 2 & 1 \end{bmatrix}.$$

Completely not related to this class: The matrix E is called a 4×4 *Latin square.* It has the property that each of the 4 symbols appear in each row and column precisely once. Can you make another 4×4 Latin square?

Story time: Two Latin squares are called orthogonal if, when you stack them one on top of each other, they give all possible pairs. For example: $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 3 & 2 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$ are orthogonal (check this). Is the second Latin square you found in the 4×4 case orthogonal to the one I gave?

Euler (an incredibly famous mathematician) could not find two 6×6 orthogonal Latin squares but he couldn't prove it. In 1901, Gaston Terry proved this by exhaustive search. *Think about it, exhaustive search in 1901!!* The number of Latin squares of order 6 is 812851200, but this can be reduced to 9408 by some tricks. So Terry, who was a school teacher, would give sets of Latin squares to his students, and their homework was to check if they were orthogonal. Eventually he finished the set of all of them, and there was the first "computer proof". This is like SETI@Home or Folding@Home: distributed computing, 1901 style.

3. Find the determinant of the matrix by whatever methods you prefer.

$$R = \begin{bmatrix} 3 & 2 & 0 & 1 \\ 4 & 0 & 1 & 2 \\ 3 & 0 & 2 & 1 \\ 9 & 2 & 3 & 1 \end{bmatrix}, \quad M = \begin{bmatrix} 3 & 1 & 2 \\ 2 & -3 & 4 \\ 2 & 8 & -4 \end{bmatrix}.$$

4. Let

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & -1 & 3 \\ 0 & 0 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 0 & 0 \\ 4 & 3 & 0 \\ 1 & 7 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 1 & -1 \\ 3 & -2 & 3 \\ 0 & -5 & 6 \end{bmatrix}.$$

Find

$$\det\left(AB^2C^T\right) + \det\left((A^T)^{-2}\left(\frac{1}{2}B\right)^3\right) - \det\left(3C^2(4A)^{-2}\right).$$