QC-LDPC codes, QC-MDPC codes and their use in post-quantum cryptography

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LDPC codes

- Low-Density Parity-Check (LDPC) codes are state-of-art forward error correcting (FEC) codes.
- Introduced by Gallager in 1962 and more recently rediscovered.
- Able to approach the channel capacity under belief propagation decoding.
- Nowadays included in many applications and standards.

- R. G. Gallager, "Low-density parity-check codes," IRE Trans. Inform. Theory, vol. IT-8, pp. 21–28, Jan. 1962.
- D. J. C. MacKay and R. M. Neal, "Good codes based on very sparse matrices," in Cryptography and Coding. 5th IMA Conference, ser. Lecture Notes in Computer Science, C. Boyd, Ed. Berlin: Springer, 1995, no. 1025, pp. 100–111.
- C. Sae-Young, G. Forney, T. Richardson, and R. Urbanke, "On the design of low-density parity-check codes within 0.0045 dB of the Shannon limit," IEEE Commun. Lett., vol. 5, no. 2, pp. 58–60, Feb. 2001.

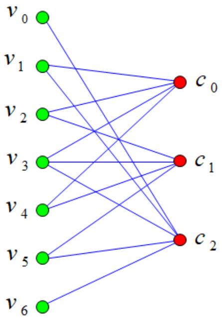
Code representation

- Binary linear block code with
  - n: code length
  - k: code dimension
  - $r = n - k$: code redundancy

Parity-check matrix

$$\mathbf{H} = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

Tanner graph



Encoding and decoding of LDPC codes

- Encoding through classical methods (e.g. generator matrix **G**).
- Efficient decoding through iterative algorithms working on the code parity-check matrix/Tanner graph.
- Soft-decision decoders:
  - Sum-product algorithm with log-likelihood ratios (LLR-SPA)
  - Min-sum algorithm and its variants (offset, weighted...)
- Hard-decision decoders:
  - Gallager's A/B algorithm
  - Bit-flipping algorithm
  - Their variants

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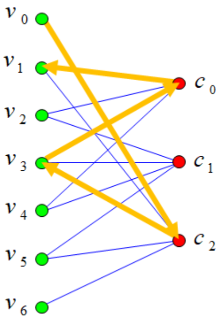

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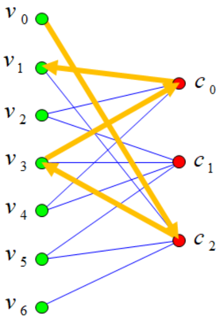

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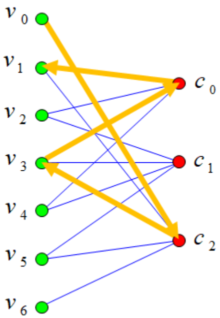
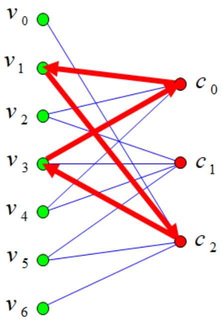


Properties affecting iterative decoding

Closed loops in the Tanner graph contaminate iterative decoders' information with correlation.



- Thus LDPC codes usually have:
  - Low density of ones in the parity check matrix
  - Few edges in the Tanner graph
  - No more than one overlapping one between any two rows/columns
  - Local cycles in the Tanner graph as long as possible
- These requirements result in LDPC codes with:
  - very small Hamming weight of the rows of $\mathbf{H}$ ($d_c \approx \log n$)
  - very long length ($n \gg 1000$)

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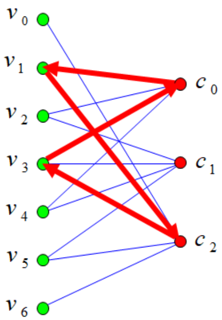

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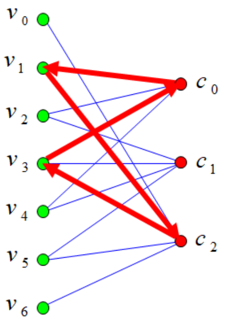

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Hard-decision decoding: Gallager's A/B

Gallager's A decoding:

1. Variable nodes send their initial value (0/1) to their neighboring check nodes.
2. Each check node c sends back to each variable node v the binary sum of all values received from its neighbours except v (marginalization).
3. Each variable node v counts the number of values received from its neighboring check nodes that disagree with its own value.
4. For each check node c, if all neighboring check nodes other than c (marginalization) disagree with the value of v, then v sends its flipped value to c, otherwise it sends its original value to c.
5. Decoding iterates from step 2, unless all parity checks are satisfied or a maximum number of iterations is reached.

Gallager's B decoding:

- Steps 1, 2, 3 and 5 like Gallager's A.
- In step 4, the value of v is flipped if the number of disagreeing check nodes except c (marginalization) exceeds a given threshold b.

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- In step 4, the value of v is flipped if the number of disagreeing check nodes except c (marginalization) exceeds a given threshold b.

Bit flipping decoding

- Similar to Gallager's B algorithm, but without marginalization.
- From [Gallager1962]:

`The decoder computes all the parity checks and then changes any digit that is contained in more than some fixed number of unsatisfied parity-check equations. Using these new values, the parity checks are recomputed, and the process is repeated until the parity checks are all satisfied.`

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Bit flipping decoding performance

- The decoding radius of LDPC codes under BF decoding cannot be determined analytically through closed form expressions.
- However, the average BF decoder performance can be estimated through a probabilistic model (under some assumptions).
- It allows computing a threshold for the number of errors for which BF converges to the right codeword in asymptotic conditions.
- It can be adapted to modeling the BF decoder performance in code-based cryptosystems.

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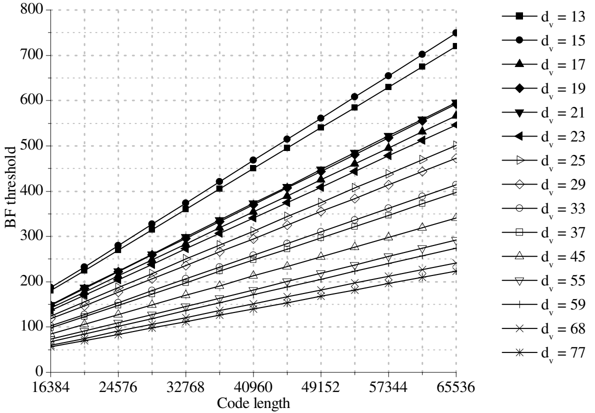
Bit flipping decoding performance - examples



BF decoding thresholds versus code length (n) for LDPC codes with code rate 3/4 and several parity-check matrix column weights (d_v).

Moderate-density parity-check (MDPC) codes

- Special case of LDPC codes with density larger than usual ($d_c \approx \sqrt{n}$).
- Mostly used in code-based cryptosystems.
- The density of their parity-check matrices/Tanner graphs does not allow avoiding short cycles.
- However, they can still be decoded through iterative decoders.

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QC-LDPC codes

- A linear block code is a quasi-cyclic (QC) code if:
  - its dimension and length are multiple of an integer p ($k = k_0 p, n = n_0 p$),
  - every cyclic shift of a codeword by n_0 positions yields another codeword.
- The generator and parity-check matrices of a QC code can assume two forms:
  - Circulant of blocks.
  - Block of circulants.

Advantage

The QC structure allows to represent the generator and parity-check matrices in a compact way (each circulant is completely described by its first row).

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Example of QC-(almost)LDPC code

$$
\mathbf{H} = \left[\begin{array}{ccccccccccc|ccccccccccc}
1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
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0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\
1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1
\end{array}\right]
$$

- Number of ciculant blocks: $n_0 = 2$.
- Code rate: $R = \frac{n_0 - 1}{n_0} = 1/2$.
- Parity-check matrix column weight: $d_v = 3$.
- Parity-check matrix row weight: $d_c = n_0 d_v = 6$.

QC-LDPC and QC-MDPC codes

$$\mathbf{H} = \left[\begin{array}{ccccccccccc|ccccccccccc}
\mathbf{1} & \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} \\
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 & & & & & \vdots & & & & & & & & & & & \vdots & & & & &
\end{array}\right]$$

- The parity-check matrix is described by its first row.
- The storage size increases linearly in the code length.
- The code length is usually very large ($10'000 \lessapprox n \lessapprox 100'000$)
- QC-LDPC codes usually have $d_c \approx \log n$.
- QC-MDPC codes usually have $d_c \approx \sqrt{n}$.

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Quantum computing

- Computer using quantum mechanics phenomena, such as quantum superposition and quantum correlation for performing calculations.
- Theorized by Richard Feynman and Yuri Manin in the early 1980s.
- Shor's algorithm (1994):
  - factorizes integers on a quantum computer,
  - given an integer N, it factors it in a time polynomial in $\log(N)$,
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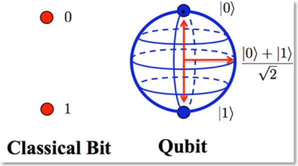

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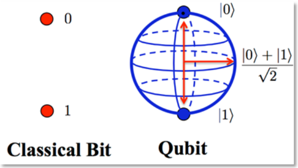

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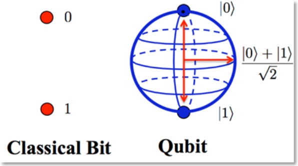

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- It must be kept at a very low temperature and isolated from any form of electromagnetic noise.
- Quantum equivalent of the first computers of the 1950s and 1960s.
- Simulators and software models available for programming.

Towards practical quantum computers (3)

- On January 2019 IBM announced Q System One, the first commercial quantum computer.
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Quantum supremacy

- Google and IBM are competing towards achieving quantum supremacy.
- The 72-qubit system that Google was developing in 2017 proved too difficult to control.
- Google then started the development of a 53-qubit system called Sycamore.
- In October 2019, Google claimed that the Sycamore processor was able to perform a calculation in 200 seconds that would have taken the world's most powerful supercomputer 10,000 years.
- IBM disclaimed this, stating that Google's system is specialized to solve a single problem, differently from IBM's general-purpose quantum computer.

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Quantum-vulnerable cryptography

The most widespread cryptographic systems today are based on mathematical problems that can be solved with Shor's algorithm:

- **RSA**: public key cryptosystem based on integer factorization (used in SSL/TLS, online banking, ATM, ...).
- **ElGamal**: public key cryptosystem based on discrete logarithm (used in SSL/TLS, ...).
- **DSA**: digital signature algorithm based on discrete logarithm (used in SSL/TLS, ...).
- **Diffie-Hellman**: key exchange protocol based on discrete logarithm (used in SSL/TLS, NFC, contactless payments, ...).
- **ECDH**: Elliptic-curve Diffie–Hellman, used for end-to-end encryption (Signal, WhatsApp, Facebook Messenger, Skype, ...).
- **ECDSA**: Elliptic-curve digital signature algorithm (used in Bitcoin (secp256k1), Ethereum, ...).

Post-quantum cryptography

Asymmetric schemes:

- Based on lattices
- Based on codes
- Based on multivariate polynomials
- Based on hash functions
- Others (isogenies ...)

Symmetric schemes:

- Symmetric encryption schemes (AES ...)
- Hash functions (SHA ...)
- Can still be used as long as Grover's algorithm is taken into account

NIST PQcrypto Project

- **NIST** has initiated a process for the development and standardization of one or more public-key cryptographic algorithms to enrich:

  - Recommendation FIPS 186-4 (Digital Signature Standard - DSS)
  - Special publication SP 800-56A Rev 2 (key establishment systems based on discrete logarithm)
  - Special publication SP 800-56B (key establishment systems based on integer factorization)

NIST PQcrypto call timeline

- **24-26 February 2016**: Announcement and description of the NIST call.
- **28 April 2016**: NISTIR 8105 report on post-quantum cryptography released.
- **20 December 2016**: Official publication of the call.
- **30 November 2017**: Deadline for submission of candidates (82 submissions).
- **30 January 2019**: Second round admission announced (26 candidates).
- **22 July 2020**: Third round admission announced (7 finalists + 8 alternate candidates).

Security level goals

NIST target (for categories 1, 3, and 5)

Computational effort required on either a classical or a quantum computer to break the AES with a key size of λ bits, $\lambda \in \{128, 192, 256\}$, through an exhaustive key search.

- On a classical computer we have complexity $2^{\lambda} C_{AES}$, where C_{AES} is the binary cost of AES.
- The quantum cost can be estimated taking into account Grover's algorithm and counting the strictly needed Clifford and T gates (which are the most expensive).

NIST Category	AES Key Size (bits)	Classical Cost (binary operations)	Quantum Cost (quantum gates)
1	128	$2^{128} \cdot 2^{14} \cdot 3 = 2^{143.5}$	$1.16 \cdot 2^{81}$
3	192	$2^{192} \cdot 2^{14} \cdot 4 = 2^{208}$	$1.33 \cdot 2^{113}$
5	256	$2^{256} \cdot 2^{14} \cdot 5 = 2^{272.3}$	$1.57 \cdot 2^{145}$

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McEliece cryptosystem

- Proposed by Robert McEliece in 1978.
- Irreducible Goppa codes were used in the original proposal.
- Secret irreducible Goppa code:
  - irreducible polynomial of degree t over $GF(2^m)$,
  - length (maximum): $n = 2^m$,
  - dimension: $k \geq n - t \cdot m$,
  - correction capability: t errors.

Robert J. McEliece
(May 21, 1942 – May 8, 2019)

Rationale

1. The number of irreducible polynomials of degree t over $GF(n)$ is $\approx n^t/t$.
2. The probability that a random polynomial is irreducible is $\approx 1/t$, and a fast algorithm exists for testing irreducibility.

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McEliece cryptosystem - key generation

Private key

- $k \times n$ generator matrix $\mathbf{G}$ of a secret Goppa code,
- random dense $k \times k$ non-singular "scrambling" matrix $\mathbf{S}$,
- random $n \times n$ permutation matrix $\mathbf{P}$.

Public key

$$\mathbf{G}' = \mathbf{S} \cdot \mathbf{G} \cdot \mathbf{P}$$

- The public code is permutation equivalent to the secret code.
- Security relies on the hardness of decoding a random-like code.

▶ E. Berlekamp, R. McEliece and H. van Tilborg, "On the inherent intractability of certain coding problems," IEEE Trans. Inf. Theory, vol. 24, no. 3, pp. 384–386, May 1978.

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McEliece cryptosystem - encryption

1. Alice gets Bob's public key $\mathbf{G}'$.
2. She generates a random error vector $\mathbf{e}$ of length n and weight t.
3. She encrypts any k-bit block $\mathbf{u}$ as

$$\mathbf{x} = \mathbf{u} \cdot \mathbf{G}' + \mathbf{e} = \mathbf{c} + \mathbf{e}$$

Alert

This only provides semantic security!

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McEliece cryptosystem - decryption

1. Bob computes

$$\begin{aligned} \mathbf{x}' &= \mathbf{x} \cdot \mathbf{P}^{-1} = \\ &= (\mathbf{u} \cdot \mathbf{S} \cdot \mathbf{G} \cdot \mathbf{P} + \mathbf{e}) \cdot \mathbf{P}^{-1} = \\ &= \mathbf{u} \cdot \mathbf{S} \cdot \mathbf{G} + \mathbf{e} \cdot \mathbf{P}^{-1} \end{aligned}$$

2. Bob decodes the secret code and obtains

$$\mathbf{u}' = \mathbf{u} \cdot \mathbf{S}$$

3. Bob computes $\mathbf{u} = \mathbf{u}' \cdot \mathbf{S}^{-1}$.

Niederreiter cryptosystem - key generation

Private key

- $r \times n$ parity-check matrix $\mathbf{H}$ of a secret code,
- random dense $r \times r$ non-singular "scrambling" matrix $\mathbf{S}$.

Public key

$$\mathbf{H}' = \mathbf{S} \cdot \mathbf{H}$$

H. Niederreiter, "Knapsack-type cryptosystems and algebraic coding theory," Problems of Control and Information Theory, 15(2):159–166, 1986.

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$$\mathbf{x} = \mathbf{H}' \cdot \mathbf{e}^T = \mathbf{S} \cdot \mathbf{H} \cdot \mathbf{e}^T$$

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Niederreiter cryptosystem - decryption

1. Bob computes
$$\mathbf{x}' = \mathbf{S}^{-1} \cdot \mathbf{x} = \mathbf{H} \cdot \mathbf{e}^T$$
2. Bob performs syndrome decoding of the secret code and obtains $\mathbf{e}$ from $\mathbf{x}'$.
3. He demaps $\mathbf{e}$ into the corresponding secret message block.

McEliece/Niederreiter cryptosystems

- GRS codes originally used in Niederreiter were attacked.
- But Goppa codes resisted cryptanalysis for more than 40 years.
- These systems are faster than competing solutions...
- ...but they require large public keys (56 KiB or more for 80-bit security).
- Attacks based on distinguishers pose some threats on high rate Goppa codes.
- They also invalidate all existing McEliece cryptosystem security proofs for high rate Goppa codes.

- D. J. Bernstein, T. Lange, and C. Peters, "Attacking and defending the McEliece cryptosystem," in *Post-Quantum Cryptography*, vol. 5299 of Springer LNCS, pp. 31–46, 2008.
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Alternatives to Goppa codes (Hamming metric)

Goppa codes
[McEliece78]

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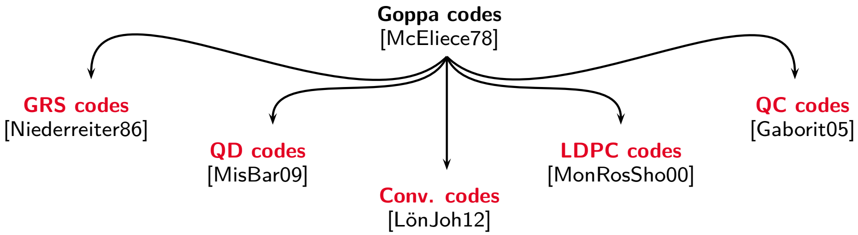

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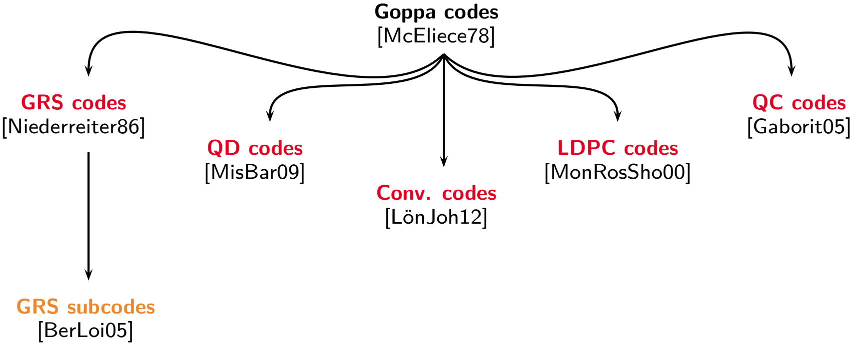

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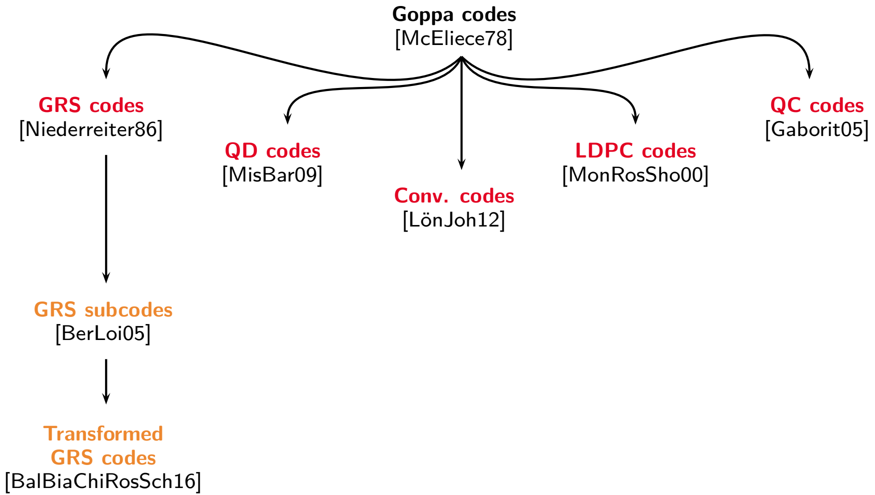

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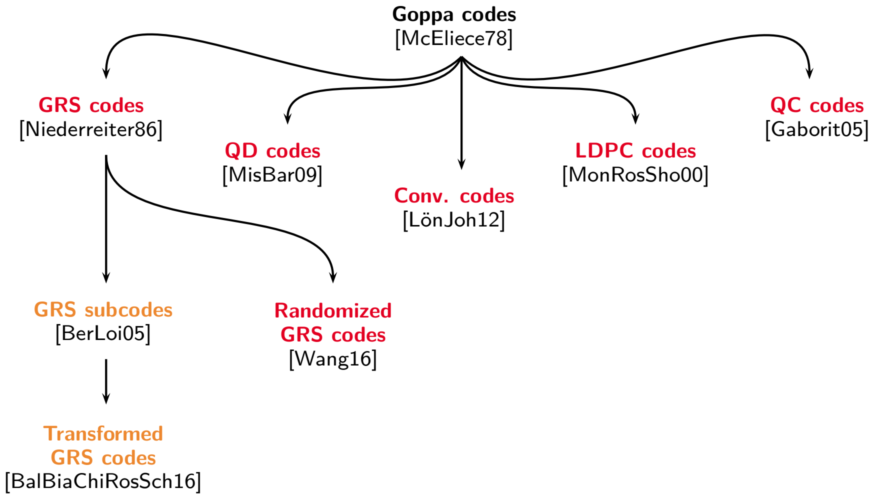

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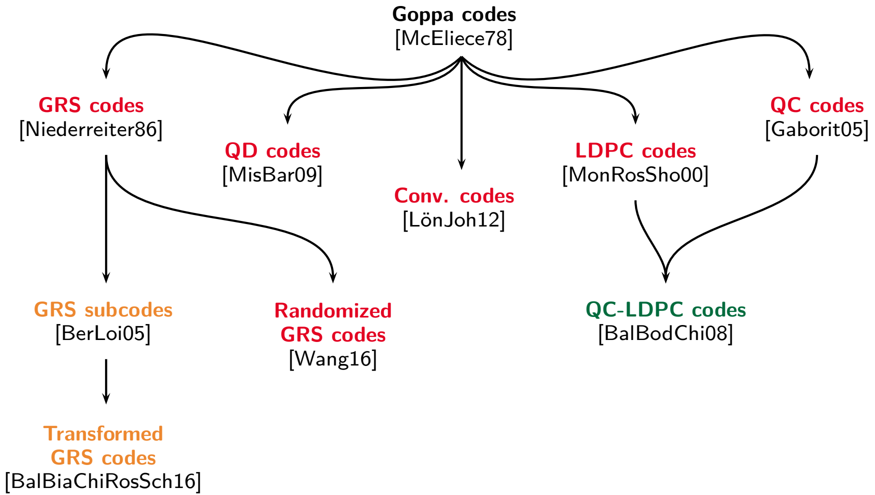

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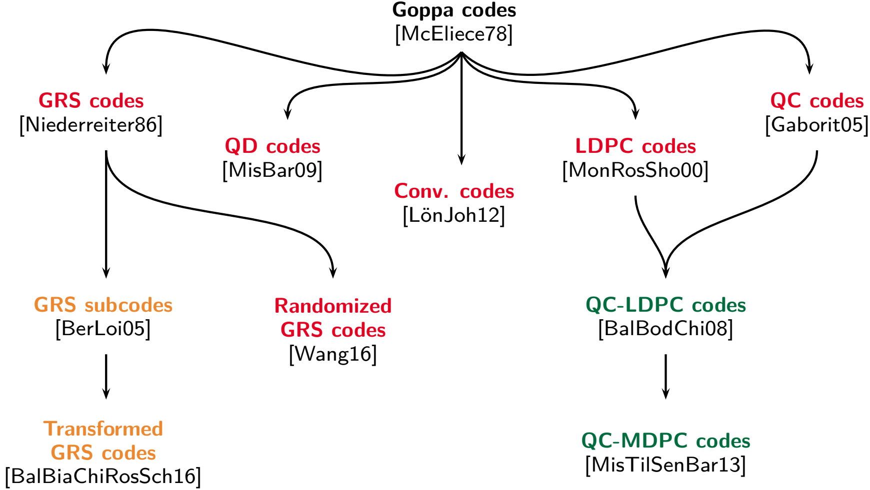

QC-LDPC code-based cryptosystems

- QC-LDPC codes bring important advantages in the framework of McEliece/Niederreiter cryptosystems:
  - The sparsity of their matrices enables very efficient decoding.
  - Quasi-cyclicity enables very compact keys.
- quasi-cyclic low-density parity-check (QC-LDPC) code-based systems introduced in 2008.
- quasi-cyclic moderate-density parity-check (QC-MDPC) code-based variants introduced in 2013.

- M. Baldi, M. Bodrato, F. Chiaraluce, "A new analysis of the McEliece cryptosystem based on QC-LDPC codes", Proc. SCN 2008, vol. 5229 of LNCS, pp. 246–262, 2008.
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Private QC-LDPC code

The private code is a QC-LDPC code with:

- rate $R = \frac{n_0-1}{n_0}$ (with $n_0 = 2, 3, 4$),
- redundancy r (in the order of some thousands),
- length $n = n_0 \cdot r$,
- dimension $k = (n_0 - 1) \cdot r$.

Secret QC-LDPC matrix

$$\mathbf{H} = [\mathbf{H}_0 | \mathbf{H}_1 | \ldots | \mathbf{H}_{n_0-1}]$$

- Each $\mathbf{H}_i$ is an $r \times r$ circulant matrix.
- Prime values for r must be chosen to avoid folding attacks.

▶ M. Koochak Shooshtari, M. Ahmadian-Attari, T. Johansson and M. Reza Aref, "Cryptanalysis of McEliece cryptosystem variants based on quasi-cyclic low-density parity check codes," in IET Information Security, vol. 10, no. 4, pp. 194-202, 2016.

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M. Koochak Shooshtari, M. Ahmadian-Attari, T. Johansson and M. Reza Aref, "Cryptanalysis of McEliece cryptosystem variants based on quasi-cyclic low-density parity check codes," in IET Information Security, vol. 10, no. 4, pp. 194-202, 2016.

Private QC-LDPC code

The private code is a QC-LDPC code with:

- rate $R = \frac{n_0 - 1}{n_0}$ (with $n_0 = 2, 3, 4$),
- redundancy r (in the order of some thousands),
- length $n = n_0 \cdot r$,
- dimension $k = (n_0 - 1) \cdot r$.

Secret QC-LDPC matrix

$$\mathbf{H} = [\mathbf{H}_0 | \mathbf{H}_1 | \dots | \mathbf{H}_{n_0 - 1}]$$

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Private QC-MDPC code

- Pick an $n \times n$ sparse matrix $\mathbf{Q}$ with average row/column weight $1 \leq m \ll n$.
- $\mathbf{Q}$ is in QC form, i.e., formed by $n_0 \times n_0$ circulant blocks.

Secret QC-MDPC matrix

$$\mathbf{H}' = \mathbf{H} \cdot \mathbf{Q} = \left[\mathbf{H}'_0 | \mathbf{H}'_1 | \ldots | \mathbf{H}'_{n_0-1}\right]$$

- $\mathbf{H}'$ has density m times greater than $\mathbf{H}$, thus it describes a QC-MDPC code.
- QC-LDPC code-based systems pick a random $\mathbf{H}$, a random $\mathbf{Q}$ and $\mathbf{H}' = \mathbf{H} \cdot \mathbf{Q}$ as private key.
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Public code

Public key

Systematic **G** for the private QC-MDPC code:

$$\mathbf{G} = \left[\mathbf{I} \quad \begin{array}{c} \left(\mathbf{H'}^{-1}_{n_0-1} \cdot \mathbf{H'}_0\right)^T \\ \left(\mathbf{H'}^{-1}_{n_0-1} \cdot \mathbf{H'}_1\right)^T \\ \vdots \\ \left(\mathbf{H'}^{-1}_{n_0-1} \cdot \mathbf{H'}_{n_0-2}\right)^T \end{array} \right]$$

- **G** is dense:
  - It allows deriving dense parity-check matrices, which are unsuitable for iterative decoding.
  - Retrieving a sparse parity-check matrix requires an unfeasible computational effort.
- **G** can be in systematic form (owing to CCA2 secure conversion):
  - Public key size $= (n_0 - 1) \cdot r$ bits.

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Encryption

- Alice has to encrypt a k-bit vector $\mathbf{u}$.
- She fetches Bob's public key $\mathbf{G}$.
- She generates a random binary intentional error vector $\mathbf{e}$ with weight t.

Encryption map

$$\mathbf{x} = \mathbf{u} \cdot \mathbf{G} + \mathbf{e} = \mathbf{c} + \mathbf{e}$$

- $\mathbf{c}$ is a codeword in the public code.
- Addition is modulo-2.
- Hence, all intentional errors are bit flipping errors.

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- To recover $\mathbf{u}$ from $\mathbf{x}$, Bob first computes the syndrome $\mathbf{s}$ through the secret QC-MDPC matrix.

Syndrome computation

$$\mathbf{s} = \mathbf{H}' \cdot \mathbf{x}^T = \mathbf{H}' \cdot (\mathbf{c} + \mathbf{e})^T = \mathbf{H}' \cdot \mathbf{e}^T$$

$$\mathbf{s} = (\mathbf{HQ}) \cdot \mathbf{e}^T = \mathbf{H} \cdot (\mathbf{eQ}^T)^T = \mathbf{H} \cdot \mathbf{e}'^T$$

- $\mathbf{e}' = \mathbf{eQ}^T$ is a binary vector with weight $\leq t' = tm$.
- From $\mathbf{s}$ Bob can recover:
  - $\mathbf{e}$ by decoding through the secret QC-MDPC $\mathbf{H}'$, or
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QC-LDPC/MDPC codes in the NIST contest

- LEDAcrypt (Low-dEnsity parity-check coDe-bAsed cryptographic systems), providing:
  - A Niederreiter-based KEM with IND-CPA and ephemeral keys.
  - A Niederreiter-based KEM with IND-CCA2 and long-term keys.
  - A McEliece-based PKC with IND-CCA2.
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Main attacks

Decoding attacks

Aimed at decrypting one or more ciphertexts without knowing the private key.

- An information set decoding (ISD) algorithm can be exploited to perform decoding of the public code.

Key recovery attacks

Aimed at recovering the private key from the public key.

- For any linear code, the rows of the parity-check matrix are codewords of its dual code.
- For QC-LDPC and QC-MDPC codes, these rows have low weight and can be searched through ISD algorithms.
- The quantum speedup due to Grover's algorithm must be taken into account.

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Reaction attacks

Observations

1. Iterative decoding algorithms do not have a deterministic decoding radius, which entails a non-zero decoding failure rate (DFR).
2. The DFR depends on the structure of the secret matrix.
3. Eve can estimate the DFR by observing Bob's reactions.

- In the CPA case, Eve performs many encryptions with suitably chosen error vectors and observes Bob's reactions during decryption.
- In the CCA2 case, the error vectors cannot be chosen by Eve, who must exploit those resulting from each encryption to make her deductions.

Countermeasure

Make the DFR negligible (i.e., $\leq 2^{-\lambda}$).

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CCA security and δ-correctness

- QC-LDPC and QC-MDPC code-based cryptosystems alone provide semantic security only.
- According to [HHK2017], a KEM can be built having IND-CCA2 reduced to the OW-CPA security of the underlying deterministic public key cryptosystem.
- It is required that the decryption failure probability of the OW-CPA scheme is $\delta \leq 2^{-\lambda}$.

δ-correctness

Probability that a (possibly unbounded) adversary is able to induce a decryption failure on a valid ciphertext, taken as the **maximum** over all possible plaintexts, and averaged over all the keypairs.

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Decoding failure probability of a given code (i.e., keypair) **averaged** over all the possible plaintexts (i.e., error vectors).

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Iterative decoders are algorithmic $\Rightarrow$ no closed form formula for their error correction capability.

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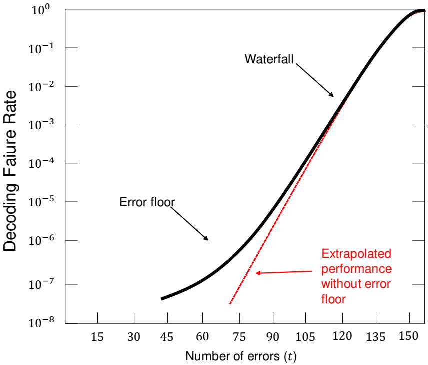
DFR extrapolation?

- One way to extend DFR curves is by extrapolation.
- This is the approach followed in BIKE.
- However, the curve slope may change (error floor) due to:
  - code structural properties,
  - code representation properties,
  - decoding algorithm properties.
- Hence, performance curves can hardly be extrapolated outside the simulated region.



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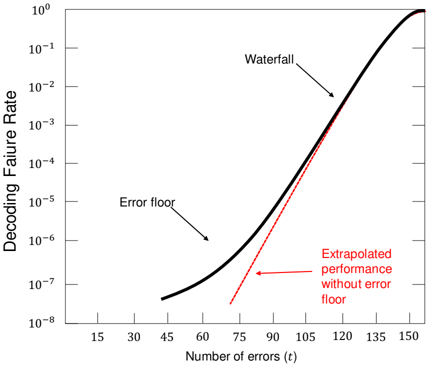

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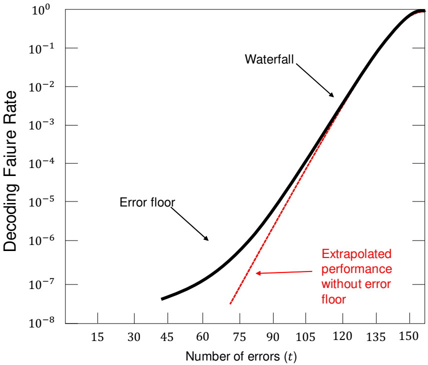

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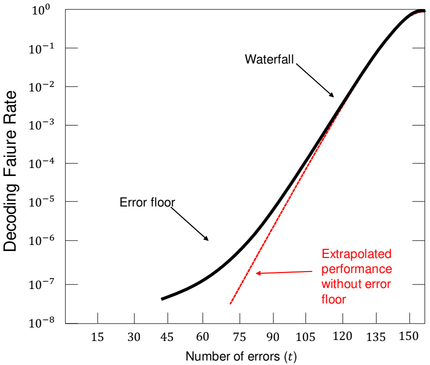

LEDAcrypt

- Suite of low-density parity-check code-based cryptosystems.
- Among the 26 second round candidates of the NIST pqcrypto competition.
- Proposing team:
  - Marco Baldi (Univpm, Italy)
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- Official website (`https://www.ledacrypt.org/`):
  - First and second round specifications.
  - Full ANSI-C99 codebase.
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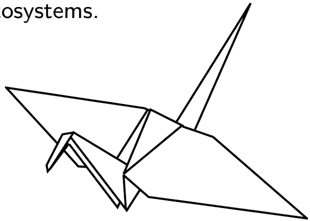

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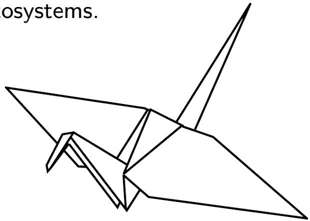

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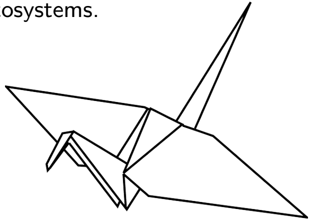

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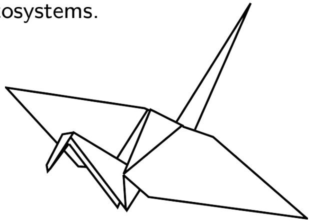

LEDAcrypt features

1. Both KEM and PKC modes.
2. Closed-form upper bound on the DFR.
3. Solves the mismatch between δ-correctness and DFR:
   - Mechanism to prevent the adversary from exploiting the advantage coming from the selection of input messages/error vectors.
   - This makes the maximum probability of a decryption failure over all plaintexts equal to the average failure probability over all plaintexts.
4. Algorithmic approach to the design of parameter sets.
5. Instances with:
   - Ephemeral keys and a DFR in the order of 10^{-9}, or
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LEDAcrypt parameters

Table: Key, ciphertext and transmitted data sizes for **LEDAcrypt-KEM-CPA** instances with ephemeral keys.

NIST Category	n_0	p	d_v	t	Private Key (B)	Public Key(B)	Ciphertext (B)	Shared Secret (B)	Transmitted data (B)
1	2	10,883	71	133	1,160	1,368	1,392	32	2,760
	3	8,237	79	84	1,920	2,064	1,056	32	3,120
	4	7,187	83	67	2,680	2,712	928	32	3,640
3	2	21,011	103	198	1,680	2,632	2,664	48	5,296
	3	15,373	117	125	2,840	3,856	1,960	48	5,816
	4	13,109	123	99	3,968	4,920	1,672	48	6,592
5	2	35,339	137	263	2,232	4,424	4,464	64	8,888
	3	25,603	155	166	3,760	6,416	3,248	64	9,664
	4	21,611	163	132	5,256	8,112	2,744	64	10,856

BIKE

- BIKE parameters:

Security	r	w	t	DFR
Level 1	12,323	142	134	2^{-128}
Level 3	24,659	206	199	2^{-192}

- Private key, public key and ciphertext sizes (in bits):

Quantity	Size	Level 1	Level 3
Private key	$\ell + w \cdot \lceil \log_2(r) \rceil$	$2,244$	$3,346$
Public key	r	$12,323$	$24,659$
Ciphertext	$r + \ell$	$12,579$	$24,915$

Weak keys in LEDAcrypt

- Presented by Daniel Apon, Corbin McNeill, Ray Perlner and Angela Robinson at the 2019 Quantum Cryptanalysis Dagstuhl Seminar.
- Leverage the product structure of the public code parity-check matrix ($\mathbf{H}' = \mathbf{HQ}$).

Rationale

Making guesses separately on $\mathbf{H}$ and $\mathbf{Q}$ (and projecting them onto $\mathbf{H}'$) accelerates ISD with respect to making them directly on $\mathbf{H}'$.

- One key is weak if
  - Occurs with probability 2^{-x}.
  - Requires the equivalent of 2^y AES operations for ISD.
  - $x + y < \lambda$, being λ the claimed security level.

▶ D. Apon, R. Perlner, A. Robinson, and P. Santini, "Cryptanalysis of LEDAcrypt," Cryptology ePrint Archive, Report 2020/455, 2020.

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Weak key examples

$n_0 = 2$, cat. 5, IND-CPA

$x \approx 44$, $y \approx 52$

$n_0 = 4$, cat. 1, IND-CPA

$x \approx 40$, $y \approx 50$

- This attack works well when:
  - n_0 is small and
  - the weights of $\mathbf{H}$ and $\mathbf{Q}$ are well balanced.
- Countermeasures:
  - increase n_0,
  - choose unbalanced weights for $\mathbf{H}$ and $\mathbf{Q}$.
- Cautious choice: $\mathbf{Q} = \mathbf{I}$ and $\mathbf{H}' = \mathbf{H}$.

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Open research challenges

We still need to work on:

- Weak keys deriving from the product structure and their countermeasures
- Iterative decoders and their theoretical modeling (DFR)
- Cryptanalysis exploiting sparse and structured matrices
- QC-LDPC and QC-MDPC code-based signature schemes

End of presentation

Thank you!

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