MATH 6101G TOPICS IN ALGEBRA (DIFFERENCE SETS), FALL 2019

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Lectures: Tuesdays and Thursdays 8:30 - 10:00am in CB 3208. Lectures begin on Thursday September 5, 2019 and end on Thursday December 5, 2019.

We may discuss alternative timeslots in the first week of classes in order to accommodate everyone's schedule.

Office hours: TBA

Textbook: Difference Sets: Connecting Algebra, Combinatorics, and Geometry, by Emily H. Moore and Harriet S. Pollatset, Student Mathematical Library, Volume 67, American Mathematical Society. Copies are available in the university bookstore.

Grading scheme:

1. **Assignments (40**%): There will be four assignments to be handed in during the term. The exercises may contain some computations requiring the use of a programming language.

2. **Presentation (20%):** Students will select up to three research papers of their choice (relating to difference sets and/or their applications) and consult with me for approval. Each student will study their chosen paper(s) in depth and prepare a presentation of approximately 30 minutes to the class. Presentations must include an introduction/overview of the chosen paper(s), background content, why the paper(s) is/are of interest, an in-depth explanation of the research contribution of the paper(s), and possible extensions of the work.

   Students will prepare electronic slides and e-mail them to me one day before their presentation. The grading will be based on the student's overall presentation quality and their ability to answer questions from the instructor and students.

   Presentations will take place during the last three weeks of classes.

3. **Final Exam (40%):** There will be a take-home final exam, details of which will be announced in class.

Academic accommodations: Students who would like to request academic accommodations due to religious obligations, pregnancy or disabilities should inform the instructor as soon as possible.

Difference sets are interesting combinatorial objects which connect number theory, combinatorics, algebra, representation theory, and geometry. Difference sets have a wide range of applications in engineering, computer science, communications, error correcting codes, cryptography, designs, and quantum information theory. Difference sets were first introduced by Singer in 1938 when he discovered the first class of difference sets with Singer parameters.
The systematic study of difference sets started in the late 1940s with the important work of Hall, who introduced the concept of multipliers. The investigation of difference sets in general groups started with the work of Bruck in 1955. Interest in the topic increased greatly when it was discovered that the existence of a difference set over a cyclic group is equivalent to the existence of a certain type of periodic sequence with good autocorrelation properties.

A difference set D in a group G is defined as follows. Let G be a group of order v, and D be a subset of G of size k. The subset D is called a (v, k, λ)-difference set if each nonzero element of G can be expressed exactly λ times as a difference of two elements of D. For instance, the set of quadratic residues {1, 2, 4} modulo 7 forms a (7, 3, 1) difference set in the additive group of the residues modulo 7 (Can you show this?). It can indeed be shown that the quadratic residues modulo a prime number p form a difference set in the additive group of the residues modulo p if and only if p is congruent to 3 modulo 4. By a simple counting argument, one can easily deduce the basic necessary condition for D to be a difference set, namely $k(k-1) = \lambda(v-1)$.

A selection of topics from the following chapters of the textbook will be covered:

Chapter 1: Introduction
Chapter 2: Designs
Chapter 3: Automorphisms of Designs
Chapter 4: Introducing Difference Sets
Chapter 5: Bruck-Ryser-Chowla Theorem
Chapter 6: Multipliers
Chapter 7: Necessary Group Conditions
Chapter 8: Difference Sets from Geometry
Chapter 9: Families from Hadamard Matrices
Chapter 10: Representation Theory
Chapter 11: Group Characters
Chapter 12: Using Algebraic Number Theory
Chapter 13: Applications

Depending on our progress, we may cover additional topics from the textbook or other resources.