3) Diagonalize $A = \begin{bmatrix} 5 & -2 \\ 1 & 3 \end{bmatrix}$ if possible.

Soln. $\det(A - \lambda I) = \begin{vmatrix} 5-\lambda & -2 \\ 1 & 3-\lambda \end{vmatrix} = (5-\lambda)(3-\lambda)+2 = \lambda^2 - 8\lambda + 17 = 0$

$$\lambda = \frac{8 \pm \sqrt{64-68}}{2} = 4 \pm i$$

$\lambda_1 = 4+i$:

$$A - (4+i)I = \begin{bmatrix} 1-i & -2 \\ 1 & -1-i \end{bmatrix} \overset{R_1 \leftrightarrow R_2}{\sim} \begin{bmatrix} 1 & -1-i \\ 1-i & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & -1-i \\ 0 & 0 \end{bmatrix}$$

$(A-(4+i)I)X = 0 \Rightarrow X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = x_2 \begin{bmatrix} 1+i \\ 1 \end{bmatrix}$ $\quad v_1 = \begin{bmatrix} 1+i \\ 1 \end{bmatrix}$

$\lambda_2 = 4-i = \bar{\lambda}_1$: $\quad v_2 = \begin{bmatrix} 1-i \\ 1 \end{bmatrix}$

$\therefore \quad D = \begin{bmatrix} 4+i & 0 \\ 0 & 4-i \end{bmatrix} \qquad P = \begin{bmatrix} 1+i & 1-i \\ 1 & 1 \end{bmatrix}$

and $\quad A = PDP^{-1}$.