

CHAPTER 1, QUESTION 28

28. Prove that $\mathbb{Z} + \mathbb{Z}\sqrt{-6}$ is not a principal ideal domain.

Solution. In a principal ideal domain every irreducible is prime (and conversely). In the domain $\mathbb{Z} + \mathbb{Z}\sqrt{-6}$ the element $\sqrt{-6}$ is irreducible (Question 27) but not prime (Question 26). Hence $\mathbb{Z} + \mathbb{Z}\sqrt{-6}$ is not a principal ideal domain. ■

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