CHAPTER 1, QUESTION 9

9. Let

$$S = \{a + bi \in \mathbb{Z} + \mathbb{Z}i \mid b \equiv 0 \pmod 2\}.$$

Is S an ideal of $\mathbb{Z} + \mathbb{Z}i$?

Solution. Clearly

$$1 + 2i \in S$$

and

$$1 + i \in \mathbb{Z} + \mathbb{Z}i.$$

However

$$(1 + i)(1 + 2i) = -1 + 3i \notin S.$$

Thus S is not an ideal of $\mathbb{Z} + \mathbb{Z}i$. ■

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