7. Use Theorem 2.3.1 to show that  $\mathbb{Z} + \mathbb{Z}\sqrt{26}$  is not Euclidean with respect to  $\phi_{26}$ .

Solution. Let

$$m = 26, p = 3, q = 7, t = 4, u = 2, r = 8.$$

Then

$$\begin{pmatrix} \frac{m}{p} \end{pmatrix} = \begin{pmatrix} \frac{26}{3} \end{pmatrix} = \begin{pmatrix} -1\\3 \end{pmatrix} = -1,$$
$$\begin{pmatrix} \frac{m}{q} \end{pmatrix} = \begin{pmatrix} \frac{26}{7} \end{pmatrix} = \begin{pmatrix} -2\\7 \end{pmatrix} = -1,$$

$$pt + qu = 3 \times 4 + 7 \times 2 = 12 + 14 = 26 = m, \ p \nmid t, \ q \nmid u,$$
  
 $r^2 \equiv 64 \equiv 12 \equiv pt \pmod{26}.$ 

Hence, by Theorem 2.3.1,  $\mathbb{Z} + \mathbb{Z}\sqrt{26}$  is not Euclidean with respect to  $\phi_{26}$ .

February 6, 2004