Chapter 7, Question 22

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22. Let K be an algebraic number field of degree n. Is it possible to find $\lambda_1, \ldots, \lambda_n \in O_K$ such that $D(\lambda_1, \ldots, \lambda_n) = -d(K)$?

Solution. Let $\{\omega_1, \ldots, \omega_n\}$ be an integral basis for K. Suppose $\lambda_1, \ldots, \lambda_n \in O_K$ are such that

$$D(\lambda_1, \ldots, \lambda_n) = -d(K).$$

As $\lambda_1, \ldots, \lambda_n \in O_K$ there exist $c_{ij} \in \mathbb{Z}$ such that

$$\begin{array}{l} \lambda_1 = c_{11}\omega_1 + \cdots + c_{1n}\omega_n, \\ . \qquad . \qquad . \qquad . \\ \lambda_n = c_{n1}\omega_1 + \cdots + c_{nn}\omega_n. \end{array}$$

Hence

$$D(\lambda_1, \ldots, \lambda_n) = (\det(c_{ij}))^2 D(\omega_1, \ldots, \omega_n).$$

Thus

$$-d(K) = (\det(c_{ij}))^2 d(K)$$

and so

$$(\det(c_{ij}))^2 = -1.$$

This is a contradiction as $\det(c_{ij}) \in \mathbb{Z}$. ∎

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