EXERCISES 9, QUESTION 4

4. Let K be an algebraic number field and O_K its ring of integers. If I is a nonzero integral ideal of O_K, prove that $I \mid< N(I) >$.

Solution. As $1 + I \in O_K/I$ and $|O_K/I| = N(I)$, we have

$$N(I)(1 + I) = 0 + I.$$

Thus

$$N(I) + I = 0 + I.$$

Thus

$$N(I) + I = 0 + I.$$

Hence

$$N(I) \in I.$$

Then

$$< N(I) > \subseteq I,$$

so that

$$I \mid< N(I) > .$$

■

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